IEooc_Application1_Exercise2: Structural decomposition of the IPAT equation

How much do individual factors contribute to change?

Sustainability, at the large scale, is about meeting both environmental and societal goals. Simply speaking, environmental sustainability means putting a cap on certain environmental impacts *I*, such as greenhouse gas emissions or emissions or particulate matter. Economic development (to a reasonable extent) is a major societal goal; it is commonly measured by the personal affluence, that is, the average spending A per person.

The following equation, called IPAT after its four constituents I, P (Population), A, and T (Technology), links the two sustainability goal indicators: the impact I with the affluence A:

$$I = P \cdot A \cdot T$$
$$[\frac{kg}{yr}] = [p] \cdot [\frac{\$}{yr \cdot p}] \cdot [\frac{kg}{\$}] \quad (1)$$

The IPAT equation is an accounting identity. It always holds, as the right side of the equation is simply a different breakdown of total impacts:

$$I = I$$
$$I = GDP \cdot \frac{I}{GDP} \quad (2)$$
$$I = P \cdot \frac{GDP}{P} \cdot \frac{I}{GDP}$$

The equation above always holds for nonzero P and GDP. The term GDP/P is the per capita affluence A, and the term I/GDP is the average environmental impact per economic output T.

In the IPAT framework, emissions have three drivers: The scale of a society, measured by its population, the personal affluence, measured by per capita spending, and the average emissions intensity of the economy, measured by impact per dollar of GDP produced.

With the IPAT equation, we can break down economy-wide change into changes in the driving factors population, affluence, and technology.

The question then is: What is the contribution of the individual factors *P*, *A*, and *T* to the change of *I*?

For a sum, Y = a + b + c + d, this question is easy to answer, just by looking at the summands. But for a product?

For small changes in the parameters, one can use the differential of the IPAT equation and the relative change:

$$\begin{aligned} I &= P \cdot A \cdot T \\ \Delta I &= \Delta P \cdot A \cdot T + P \cdot \Delta A \cdot T + P \cdot A \cdot \Delta T \\ \frac{\Delta I}{I} &= \frac{\Delta P}{P} + \frac{\Delta A}{A} + \frac{\Delta T}{T} \end{aligned} \quad (3)$$

From the last line of the equation, the relative change ΔT/T can be determined directly and converted to annual rates via the power law given above. Note: This equation is approximately true only for small changes in the parameters. Also, adding up the %-changes of the three parameters to get the relative change of I is only approximately true for small changes.

For larger and arbitrary changes in the parameters, one can calculate the following decomposition:

$$\begin{aligned} \Delta I &= P_2 \cdot A_2 \cdot T_2 - P_1 \cdot A_1 \cdot T_1 \\ \Delta I &= P_2 \cdot A_2 \cdot T_2 - P_2 \cdot A_2 \cdot T_1 + P_2 \cdot A_2 \cdot T_1 - P_1 \cdot A_1 \cdot T_1 \\ \Delta I &= P_2 \cdot A_2 \cdot T_2 - P_2 \cdot A_2 \cdot T_1 + P_2 \cdot A_2 \cdot T_1 - P_2 \cdot A_1 \cdot T_1 + P_2 \cdot A_1 \cdot T_1 - P_1 \cdot A_1 \cdot T_1 \\ \Delta I &= P_2 \cdot A_2 \cdot \Delta T + P_2 \cdot \Delta A \cdot T_1 + \Delta P \cdot A_1 \cdot T_1 \end{aligned} \quad (4)$$

Which shows how the changes in P, A, and T drive the change in I. This decomposition holds exactly, but it is not the only one:

$$\begin{aligned} \Delta I &= P_2 \cdot A_2 \cdot T_2 - P_1 \cdot A_1 \cdot T_1 \\ \Delta I &= P_2 \cdot A_2 \cdot T_2 - P_1 \cdot A_2 \cdot T_2 + P_1 \cdot A_2 \cdot T_2 - P_1 \cdot A_1 \cdot T_1 \\ \Delta I &= P_2 \cdot A_2 \cdot T_2 - P_1 \cdot A_2 \cdot T_2 + P_1 \cdot A_2 \cdot T_2 - P_1 \cdot A_2 \cdot T_1 + P_1 \cdot A_2 \cdot T_1 - P_1 \cdot A_1 \cdot T_1 \\ \Delta I &= \Delta P \cdot A_2 \cdot T_2 + P_1 \cdot A_2 \cdot \Delta T + P_1 \cdot \Delta A \cdot T_1 \end{aligned} \quad (5)$$

In total, there are six such combinations, which one can find by other permutations of the indices 1 and 2. There are three option to substitute index 1 for index 2 in the first option, and another two options for each case for the second substitution:

$$\Delta I = P_2 \cdot A_2 \cdot T_2 - P_1 \cdot A_1 \cdot T_1$$
$$\Delta I = P_2 \cdot A_2 \cdot T_2 - P_2 \cdot A_1 \cdot T_2 + P_2 \cdot A_1 \cdot T_2 - P_1 \cdot A_1 \cdot T_1$$
$$\Delta I = P_2 \cdot A_2 \cdot T_2 - P_2 \cdot A_1 \cdot T_2 + P_2 \cdot A_1 \cdot T_2 - P_1 \cdot A_1 \cdot T_2 + P_1 \cdot A_1 \cdot T_2 - P_1 \cdot A_1 \cdot T_1$$
$$\Delta I = \Delta P \cdot A_1 \cdot T_2 + P_1 \cdot A_1 \cdot \Delta T + P_2 \cdot \Delta A \cdot T_2 \tag{6}$$
$$\Delta I = P_2 \cdot A_2 \cdot T_2 - P_2 \cdot A_1 \cdot T_2 + P_2 \cdot A_1 \cdot T_2 - P_2 \cdot A_1 \cdot T_1 + P_2 \cdot A_1 \cdot T_1 - P_1 \cdot A_1 \cdot T_1$$
$$\Delta I = \Delta P \cdot A_1 \cdot T_1 + P_2 \cdot A_1 \cdot \Delta T + P_2 \cdot \Delta A \cdot T_2$$

In summary:

$$\Delta I = \Delta P \cdot A_1 \cdot T_1 + \Delta A \cdot P_2 \cdot T_1 + \Delta T \cdot P_2 \cdot A_2$$
$$\Delta I = \Delta P \cdot A_2 \cdot T_1 + \Delta A \cdot P_1 \cdot T_1 + \Delta T \cdot P_2 \cdot A_2$$

$$\Delta I = \Delta P \cdot A_2 \cdot T_2 + \Delta A \cdot P_1 \cdot T_1 + \Delta T \cdot P_1 \cdot A_2$$
$$\Delta I = \Delta P \cdot A_2 \cdot T_2 + \Delta A \cdot P_1 \cdot T_2 + \Delta T \cdot P_1 \cdot A_1 \tag{7}$$

$$\Delta I = \Delta P \cdot A_1 \cdot T_2 + \Delta A \cdot P_2 \cdot T_2 + \Delta T \cdot P_1 \cdot A_1$$
$$\Delta I = \Delta P \cdot A_1 \cdot T_1 + \Delta A \cdot P_2 \cdot T_2 + \Delta T \cdot P_2 \cdot A_1$$

All six equations are equally valid, which is why it is recommended to take the arithmetic average of these options when determining the contribution of the changes of the individual parameters to the total (Dietzenbacher & Los 1998). This breakdown and the subsequent averaging is called *structural decomposition analysis*:

$$\begin{aligned}\Delta I = &\frac{1}{6}\cdot\left(2\cdot A_1 \cdot T_1 + A_2 \cdot T_1 + A_1 \cdot T_2 + 2\cdot A_2 \cdot T_2\right)\cdot \Delta P\\ &+\frac{1}{6}\cdot\left(2\cdot P_1 \cdot T_1 + P_2 \cdot T_1 + P_1 \cdot T_2 + 2\cdot P_2 \cdot T_2\right)\cdot \Delta A\\ &+\frac{1}{6}\cdot\left(2\cdot A_1 \cdot P_1 + A_2 \cdot P_1 + A_1 \cdot P_2 + 2\cdot A_2 \cdot P_2\right)\cdot \Delta T\end{aligned} \tag{8}$$

From this, the relative change compared to I_1 can be calculated:

$$\frac{\Delta I}{I} := \frac{\Delta I}{I_1} = \frac{1}{6}\cdot\left(2\cdot A_1\cdot T_1 + A_2\cdot T_1 + A_1\cdot T_2 + 2\cdot A_2\cdot T_2\right)\cdot\frac{\Delta P}{P_1\cdot A_1\cdot T_1}$$
$$+\frac{1}{6}\cdot\left(2\cdot P_1\cdot T_1 + P_2\cdot T_1 + P_1\cdot T_2 + 2\cdot P_2\cdot T_2\right)\cdot\frac{\Delta A}{P_1\cdot A_1\cdot T_1}$$
$$+\frac{1}{6}\cdot\left(2\cdot A_1\cdot P_1 + A_2\cdot P_1 + A_1\cdot P_2 + 2\cdot A_2\cdot P_2\right)\cdot\frac{\Delta T}{P_1\cdot A_1\cdot T_1}$$

$$\frac{\Delta I}{I} = \frac{1}{6}\cdot\left(2+\frac{A_2}{A_1}+\frac{T_2}{T_1}+2\cdot\frac{A_2\cdot T_2}{A_1\cdot T_1}\right)\cdot\frac{\Delta P}{P_1}$$
$$+\frac{1}{6}\cdot\left(2+\frac{P_2}{P_1}+\frac{T_2}{T_1}+2\cdot\frac{P_2\cdot T_2}{P_1\cdot T_1}\right)\cdot\frac{\Delta A}{A_1}$$
$$+\frac{1}{6}\cdot\left(2+\frac{A_2}{A_1}+\frac{P_2}{P_1}+2\cdot\frac{A_2\cdot P_2}{A_1\cdot P_1}\right)\cdot\frac{\Delta T}{T_1} \tag{9}$$

Here, by definition, $\Delta I/I := \Delta I/I_1$, $\Delta P/P := \Delta P/P_1$, $\Delta A/A := \Delta A/A_1$, and $\Delta T/T := \Delta T/T_1$.

Task: Using the example of CO_2 emissions for the USA and China between 1990 and 2017, determine the contribution of the individual factors P, A, and T in the IPAT equation to overall change in CO_2 emissions (structural decomposition analysis of the IPAT equation)!

Use the average in equation 9 above.

Interpret your findings!

Data: In the accompanying Excel workbook, IEooc_Application1_Exercise2a_IPAT_SDA.xlsx, the relevant data shown below in Table 1 (USA and China from 1990 to 2017) are given.

Table 1: IPAT equation and results for T for the US and China for two historic dates and two 2050 scenarios. Source: IEooc_Application1_Exercise2.

IPAT	I (Mt CO_2/yr)	P (million)	A (2011 I$ p.cap)	T (kg CO_2/I$)
China 1990	3211	1172	1526	1.79
China 2017	11735	1410	15309	0.54
China 2050 min	11207	1364	55852	0.15
China 2050 max	2241	1364	55852	0.030
US 1990	6400	253	34062	0.74
US 2017	6400	324	54255	0.36
US 2050 min	3200	390	104291	0.078
US 2050 max	640	390	104291	0.016

References:

Introductory exercise to the IPAT equation in the IEooc: IEooc_Application1_Exercise2_IPAT_Equation

Link to the IPAT equation on Wikipedia: https://en.wikipedia.org/wiki/I_%3D_PAT

Jackson, T., 2009. Prosperity without growth. Routledge, London.

Erik Dietzenbacher & Bart Los (1998): Structural Decomposition Techniques: Sense and Sensitivity, Economic Systems Research, 10:4, 307-324. http://dx.doi.org/10.1080/09535319800000023