IEooc_Methods2_Exercise 2: Material Cycle, Recycling, Sensitivity Analysis, Error Propagation, Elasticities

Sample solution

Goal: *System insight into material cycles and recycling systems using the example of beverage cans in Germany. Conduct a sensitivity analysis, error propagation and calculation of result elasticities.*

In 2014, about 1.9 billion beverage cans were sold in Germany (source: Verband der Getränkedosenhersteller (BCME)). Beverage cans are made from either aluminum or tin; for this exercise we assume that all beverage cans are made from aluminum, have a capacity of 0.5 l, and weigh 16 g. It is estimated that after the introduction of can deposits (compulsory deposits for beverage containers), 96 % of all cans are returned and follow proper recycling channels. Before the introduction of can deposits, the share of cans returned was probably only around 25 %. About 95 % of the aluminum contained in the cans is reclaimed as scrap during the material recovery process and re-melted to produce new beverage cans. However, since the true quantity recovered is uncertain, an error analysis with a value of 95 ± 3 % must be conducted. During the remelting process of the drinking can scrap to produce cast bars as well as the rolling of the bars into thin metal sheets, about 3 ± 2 % of the aluminum is lost due to oxidation processes. During the stamping process of the metal sheets to produce cans, 10 % of the aluminum inputs remains as trimming waste, which is then collected and returned to the remelting process. Figure 1 below shows a typical system definition of the material cycle. The flows that are actually part of the material cycle are shown in bold. It is assumed that the all secondary aluminum is returned to the can production process. (The beverage can recycling system actually is one of only few closed loop recycling systems.)

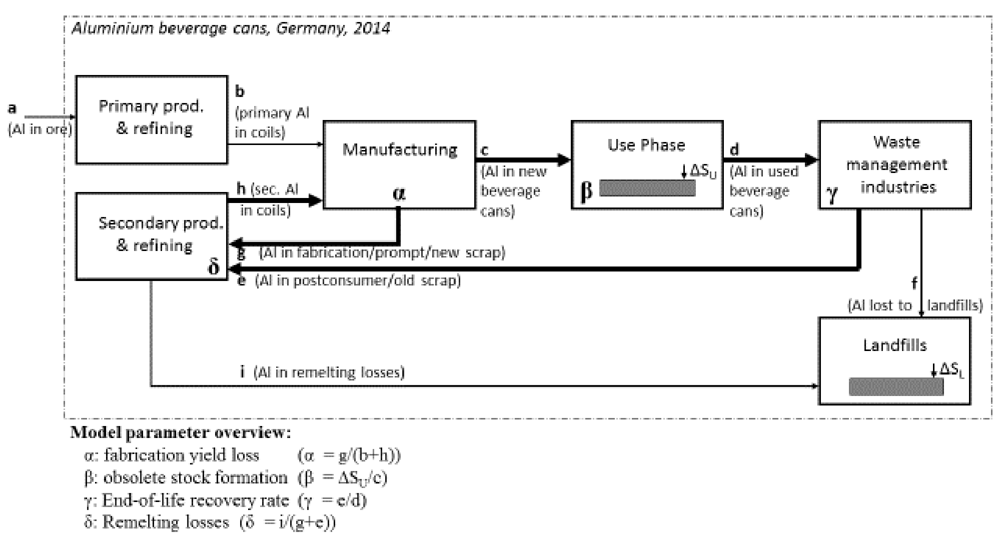

Figure 1: General system definition of the material cycle. Loss during use (ΔS_U) and in landfills (ΔS_L) are balanced through adjustments to the quantity of inflow of ore and other resources (a).

Tasks:

1) **How big are the four model parameters α-δ? What error intervals or alternative values are assigned to the four parameters?**

The definitions of the four parameters can be extracted from fig. 1. The data within the text are used accordingly.
α = 0.1. (scrap rate of fabrication). No error or alternative value given.
β = 0.04 (0.75). Rate of loss cans, present and historic value. No error given. Calculated as 1-0.96 or 1-0.25.
γ = (95 ± 3) % extraction of aluminum scrap from cans. No alternative value given.
δ = (3 ± 2) % rate of less during melting process. No alternative value given.

2) **Can the system be completely defined only using the given parameters? Give an explanation why or why not?**

The system has 9 flows, 2 inventory changes, hence 11 system variables. There are six processes which means six mass balance equations (constraints), so that five additional parameters will be required to solve all model equations. Four are known (α-δ) but this is not sufficient Hence, the answer is no. One additional piece of information is needed.

3) **Identify an analytical solution for the system from fig. 1 for Germany in 2014!**

For Germany 2014, flow c is known (c = C = 1.9e9 cans/yr * 0.016 kg/can = 30.4 kt/yr). This is the last parameter missing that we need to solve the entire system.

This will help us to come up with following solution:

c	$= C$	(external value)
$\Delta S_U = \beta \cdot c$	$= \beta \cdot C$	(defining formula for β)
$d = c - \Delta S_U$	$= (1-\beta) \cdot C$	(mass balance use phase)
$e = \gamma \cdot d$	$= \gamma \cdot (1-\beta) \cdot C$	(defining formula for γ)
$f = (1-\gamma) \cdot d$	$= (1-\gamma) \cdot (1-\beta) \cdot C$	(mass balance waste mgt.)

This was the easy part. Now we must calculate the recycling cycle.

First, we adapt the defining formula for α using the mass balance for the processing industries:

$\alpha = g/(b + h)$ $\quad b + h = g + c \quad \rightarrow \quad g = \alpha / (1-\alpha) \cdot C$

From this we receive the loss from the melting process as well as loss from disposal to landfill:

$i = \delta \cdot (g + e) \quad = \delta \cdot (\alpha / (1-\alpha) + \gamma \cdot (1-\beta)) \cdot C$ (Def.gl. for δ)

$\Delta S_L = f + i \quad = ((1-\gamma) \cdot (1-\beta) + \delta \cdot (\alpha / (1-\alpha) + \gamma\, (1-\beta)))\, C$ (Mass.bal. Dep.)

The whole volume of secondary aluminum defines as

$h = g + e - i \quad = (1-\delta) \cdot (\alpha / (1-\alpha) + \gamma \cdot (1-\beta)) \cdot C$ (Mass.bal. Sec. Prod.)

Consequently, the total volume of primary aluminum needed is:

$b = g + c - h \quad = (1 / (1-\alpha) + 1 - (1-\delta)\, (\alpha / (1-\alpha) + \gamma \cdot (1-\beta)))\, C$ (Mass.bal. SecP)

Finally, the total content of aluminum in the ore is

$a = b \quad = (1 / (1-\alpha) + 1 - (1-\delta)\, (\alpha / (1-\alpha) + \gamma \cdot (1-\beta)))\, C$ (Mass.bal PrimP)

4) What is the total loss of aluminum V in the Germany 2014 system ($V = \Delta S_U + \Delta S_L$)?

We calculate the flows ΔS_U, f and i:

ΔS_U = 1.22 kt/yr

f = 1.46 kt/yr

i = 0.93 kt/yr

This totals in 3.6 kt/yr or almost 12 % of the total volume.

5) How big is the ratio between total loss and total quantity of aluminum in beverage cans (total rate of loss r)?

$$r := \frac{V}{c} = \frac{\Delta S_U + \Delta S_L}{c}$$

3.6 kt/yr / 30.4 kt/yr = 12%.

6) What is the share of secondary aluminum in sold beverage cans (recycled content)? Is this a realistic result and why or why not?

We define the share of secondary aluminum as RC = h / (c + g)

$RC = (1-\delta) \cdot (\alpha / (1-\alpha) + \gamma \cdot (1-\beta)) \cdot (1-\alpha) = 0.89$

This value is realistic based on the organizing system for can recycling (cycle of materials clearly separated from other Al waste and scrap flows)

Nevertheless about 10 % of the aluminum used in beverage cans is lost every year.

7) What is the difference between sensitivity analysis, error analysis and the calculation of elasticities?

The difference lies with both, objectives as well as calculation methods. With the sensitivity analysis, model results for different possible parameter values are calculated. These values can differ substantially and refer to different years, scenarios, as well as conditions (before/after). In this system's case β was the variable with a mean value of 0.96 today and 0.25 prior.

An error analysis determines the deviation of modeled results from the base value based on slight changes to the parameters due to our ignorance of the true value. Hence, other than a sensitivity analysis, which determines a new base value, error analyses only determine the possible deviation from formerly defined base values that reflect our ignorance of the true parameter values.

Elasticity is a way to measure marginal changes of model results if also parameters change only marginally. These numbers that can either be used for a sensitivity analysis or an error analysis but can also be reported on its own.

8) Conduct a sensitivity analysis: first for the total loss of Al and then for the share of secondary aluminum in the sold beverage cans! Consider two parameter changes: first the historical parameter value β and second a potential increase of sold beverage cans by 50 %.

We take the formula for the flow and indicator and then calculate the values for the two given changes.

Total losses $V = \Delta S_U + \Delta S_L = \beta \cdot C + ((1-\gamma) \cdot (1-\beta) + \delta \cdot (\alpha / (1-\alpha) + \gamma \cdot (1-\beta))) \cdot C$

Secondary aluminum content $RC = (1-\delta) \cdot (\alpha / (1-\alpha) + \gamma \cdot (1-\beta)) \cdot (1-\alpha)$

	Base value	**Use Historic value f. β**	**Assume 50% increase in C**
V	3.61 kt/yr	23.5 kt/yr	5.41 kt/yr
RC	89.3%	30.4%	89.3%

Before the introduction of the can deposit the losses were very high as shown by the calculation of the historic rate of loss assuming constant consumption. The secondary aluminum share would be only 30 % given this scenario. If the consumption of cans increases given that all other parameters remain constant, the loss increases proportionally while the share of secondary aluminum remains the same.

9) Conduct an error propagation (maximal error) for the total loss V regarding parameters γ and δ! Interpret the results! Is the calculation method actually applicable for γ and δ?

We first define the partial derivation of the total loss V with respect to γ and δ and establish the formula for the maximal error error:

$$\Delta V = \left|\frac{\partial V}{\partial \gamma}\right| \cdot |\Delta\gamma| + \left|\frac{\partial V}{\partial \delta}\right| \cdot |\Delta\delta|$$

$$\frac{\partial V}{\partial \gamma} = (\delta - 1)\cdot(1-\beta)\cdot C = -28.3kt / yr$$

$$\frac{\partial V}{\partial \delta} = \left(\frac{\alpha}{1-\alpha} + \gamma\cdot(1-\beta)\right)\cdot C = 31.1kt / yr$$

$$\Delta\gamma = 0.03$$

$$\Delta\delta = 0.02$$

This results in

$$\Delta V = \left|\frac{\partial V}{\partial \gamma}\right| \cdot |\Delta\gamma| + \left|\frac{\partial V}{\partial \delta}\right| \cdot |\Delta\delta| = 0.85kt / yr + 0.62kt / yr = 1.47kt / yr$$

Rounded properly the maximal error of the total loss is 1.5 kt/yr. With this result we can come up with following properly rounded final result of the error analysis

$$V = (3.6 \pm 1.5)kt / yr$$

The relative error of 0.03/0.95 of γ is small. It makes sense to conduct a global error assessment with respect to γ since a) the change of the system is minimal and the linear approximation for the error estimation is applicable and b) there is no singularity of the derivation V with respect to the addressed parameters.

The relative error of 0.02/0.03 δ is very big (67 %!). Nevertheless, it still makes sense to conduct a maximal error assessment with respect to δ since a) the derivation V of δ is independent from δ and hence the linear approximation of the error estimation applies to any δ and b) there are no singularities of the derivation V for the addressed range of parameters.

10) **Calculate the point elasticity for the total loss ($\Delta S_U + \Delta S_L$) in Germany 2014 regarding the four parameters α-δ and the quantity of sold cans! Interpret the results!**

We provide the solutions for the absolute and relative sensitivity analyses (the latter also being termed point elasticity):

First, we calculate the partial derivation V with respect to the five parameters and determine their values, so we can calculate absolute sensitivities:

$$S(V,C)=\frac{\partial V}{\partial C}=ß+\left((1-\gamma)\cdot(1-ß)+\delta\cdot\left(\frac{\alpha}{1-\alpha}\right)+\gamma\cdot(1-ß)\right)=0.119$$

$$S(V,\alpha)=\frac{\partial V}{\partial \alpha}=\frac{\delta\cdot C}{(1-\alpha)^2}=1.13kt/yr$$

$$S(V,\beta)=\frac{\partial V}{\partial \beta}=\gamma\cdot(1-\delta)\cdot C=28.0kt/yr$$

$$S(V,\gamma)=\frac{\partial V}{\partial \gamma}=(\delta-1)\cdot(1-\beta)\cdot C=-28.3kt/yr$$

$$S(V,\delta)=\frac{\partial V}{\partial \delta}=\left(\frac{\alpha}{1-\alpha}+\gamma\cdot(1-\beta)\right)\cdot C=31.1kt/yr$$

Interpretation: the absolute sensitivities are coupling coefficients that indicate how much a system variable changes in absolute terms as results of an absolute change of the model parameter.

Example 1: C changes by 2 kt/yr: hence V changes by about 0.12 * 2 kt/yr = 0.24 kt/yr.

Example 2: γ increases by 1 %: V changes by about -28.3 kt/yr * 0.01 = -0.28 kt/yr. Since γ is getting bigger less aluminum gets lost in landfills and the losses decrease. Hence there is a negative result.

The relative sensitivities or point elasticities are:

$$\bar{S}(V,C) = \frac{\partial V}{\partial C} \cdot \frac{C}{V} = 1$$

$$\bar{S}(V,\alpha) = \frac{\partial V}{\partial \alpha} \cdot \frac{\alpha}{V} = 0.031$$

$$\bar{S}(V,\beta) = \frac{\partial V}{\partial \beta} \cdot \frac{\beta}{V} = 0.31$$

$$\bar{S}(V,\gamma) = \frac{\partial V}{\partial \gamma} \cdot \frac{\gamma}{V} = -7.5$$

$$\bar{S}(V,\delta) = \frac{\partial V}{\partial \delta} \cdot \frac{\delta}{V} = 0.26$$

General: the point elasticities show by how many % the variable changes if the parameter changes by 1 %.

Example 1: C changes by 3.75 %: Since S_bar(V,C) = 1, V also changes by 3.75 %. If V is proportional to one parameter, the relative sensitivity of V is 1 with respect to this parameter.

Example 2: α changes by 2 %: Since S_bar(V,α) = 0.031, V changes by 0.06 %. Hence an improvement of the quantity produced within the processing industries has a small impact on total losses. A bigger relative impact of parameter values on results can be seen when changing β and δ (coupling 0.31 and 0.26, respectively) and γ (increase of 95 % to 95.95 % would decrease losses by 7.5 %).