IEooc_Methods3_Exercise1: Dynamic model of the German steel cycle, 1800-2008

Sample solution

1) **Draw a graphical system definition for the consumption, use, disposal, and waste management of steel-containing products!**

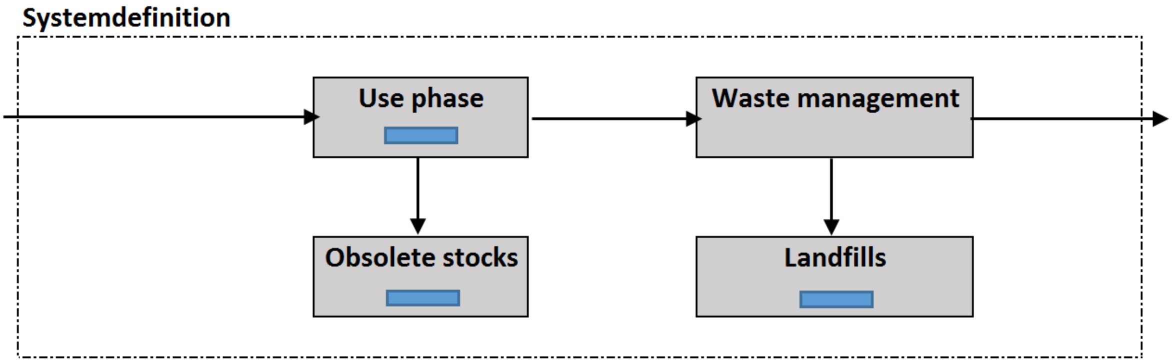

Fig. 1: System definition, steel use in Germany, 1800-2008.

2) **Use the method of dynamic stock modelling that was introduced in the lecture to estimate the in-use-stock of steel by product category as a function of time! Assume normally distributed product lifetimes. (Hint: The problem can be broken down into the following steps: a) Determine the probability distribution of a product leaving the in-use stock for a given product age (Excel-function NORM.VERT or NORM.DIST), b) Calculate, for every year and every age-cohort, the amount of steel leaving the in-use stock. (For equation see lecture, for a suggested scheme see sheet 'Stock model vehicles', Excel-function SVERWEIS or VLOOKUP), c) Calculate the total amount of steel leaving the use phase in a given year by summing up over all age-cohorts, d) Calculate the accumulation of obsolete stocks and the amount of steel scrap recovered from end-of-life products with the given parameters for obsolete stock formation and scrap recovery. Write down a mathematical equation for each calculation step!**

Step a) In the table ‚raw data', columns P-S, the probability distributions for a product leaving the stock can be filled in. The columns show the probability of a product of a certain product group of age X leaving the in-use stock. X is given in column O. To facilitate subsequent calculation the age column starts with -209 and ends at 209 years. The probability of leaving stock at negative age is of course zero. For the positive age values we use the Excel function NORM.VERT or NORM.DIST:

=NORM.VERT($O414;K$3;K$5;FALSCH/FALSE)
=NORM.VERT(age; average lifetime ; standard deviation ; FALSCH/FALSE: probability density instead of cumulative probability distribution.)

The mathematical relationship behind NORM.VERT or NORM.DIST is shown in equation 1, where PDF(t-t‘) denotes the probability density for a product leaving stock at age t-t‘. t‘ is the age-cohort and t-t‘ hence is the age of the product:

$$PDF(t-t') = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot \exp\left(-\frac{(t-t'-\mu)^2}{2\sigma^2}\right) \qquad (1)$$

Note: As the normal distribution is continuous but we are working with discrete time steps small deviations from the exact result will occur. For some equations the discrete time step Δt (= 1 year) needs to be written in the equation so that the unit becomes correct. The unit of the PDF is 1/yr (because of. 1/σ). As an example, Fig. 2 shows the probability distribution for the obsolescence of steel in buildings.

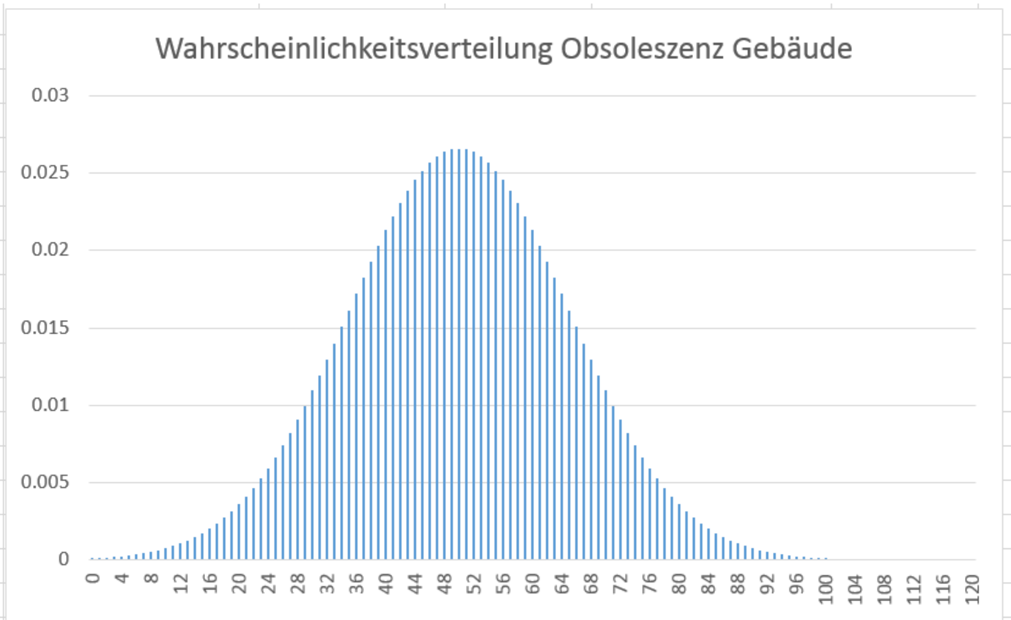

Fig. 2: Probability distribution for the end-of-life/obsolescence of steel in buildings and infrastructure in Germany. X-Axis: Age in years.

Step b) We create four new sheets, one for each product group. On each of these sheets we are going to calculate the flows of products leaving the use phase. On each sheet we create an empty table with the years 1800-2008 as row indices and age-cohorts 1800-2008 are column indices. Steel stocks and flows before 1800 are neglected.

For each combination of year and age-cohort t and t‘, we calculate the flow of end-of-life products of product category k and age-cohort t‘ in the year t, O(t,t‘,k) as

$$O(t,t',k) = PDF(t-t') \cdot I(t',k) \cdot \Delta t$$

Δt is there to get the unit right. We use the SVERWEIS/VLOOKUP function to help Excel find the right numbers, first, to get the right historical inflow I(t‘,k) and second, to get the right value for the PDF:

= SVERWEIS(K$2;Raw data!$A$3:$E$211;2) *SVERWEIS(($B22-K$2);Raw data!$O$3:$S$421;2)
= SVERWEIS(t‘;table with inflow data; Column 2) *SVERWEIS((t-t‘); table with PDF; Column 2)

Step c) The column sum of all flow values in the new table indicates the total amount of steel that has already left the age-cohort. The row sum indicates the total amount of end-of-life steel O(t) of all age-cohorts t‘ within the year t.

$$O(t,k) = \sum_{t'} O(t,t',k) = \sum_{t'} PDF(t-t') \cdot I(t',k) \cdot \Delta t$$

The above calculation is repeated on each of the four sheets for the four product groups.

Step d) The flows of steel into obsolete steel stocks are the result of the multiplication of the outflow with the obsolete stock formation parameter o(k) (Raw data J7-M7).

$$obs(t,k) = O(t,k) \cdot o(k)$$

The complement, O(t,k)·(1-o(k)), gets sent to the waste management industries. A fraction of this flow is not recovered as steel scrap but end up in landfills, (columns AH-AK), the rest is called old or postconsumer scrap and is available for recycling (columns AM-AP). With the old scrap recovery rate γ(k) (Raw data, J9-M9) the following formulae hold:

$$\text{PostconsumerScrap}(t,k) = \gamma(k) \cdot O(t,k) \cdot \left(1 - o(k)\right)$$

$$\text{SteelToLandfill}(t,k) = \left(1 - \gamma(k)\right) \cdot O(t,k) \cdot \left(1 - o(k)\right)$$

The overall amount of cumulative obsolete stocks and landfill stocks is obtained by summing up, results shown in (AC214, AH214): Here, t‘ is just a summation index and not the cohort index.

$$Obsolete\_Stocks\_2008 = \sum_{k,t'=1800}^{t'=2008} O(t',k) \cdot o(k)$$

$$Landfills\_2008 = \sum_{k,t'=1800}^{t'=2008} \left(1 - \gamma(k)\right) \cdot O(t',k) \cdot \left(1 - o(k)\right)$$

3) **Evaluate and discuss your results! How large were the per capita steel stocks in the four product groups in Germany in 1850, 1900, 19040, 1970, and 2008? How large were steel stocks in Germany in 2008, total, per product group, and per capita? What is the ratio between steel stocks and steel consumption (unit?)? How do steel in-use-stocks relate to the obsolete steel stocks and steel stocks in landfills?**

To obtain the total steel stocks we copy into a new sheet ‚Total stock' the final steel consumption (columns B-E) and the results for O(t) (columns G-J). We then calculate the annual stock change via

$$\Delta S(t,k) = I(t,k) - O(t,k)$$

(columns L-O). Then we calculate the total stocks by product group by

$$S(t,k) = \sum_{t'} \Delta S(t',k)$$

Here, t' is just a summation index and not the cohort index. The results are shown in columns Q-T, column U contains the total stock S(t):

$$S(t) = \sum_{k} S(t,k)$$

The values for S(t) are then divided by the total population P(t) in order to obtain the per capita stocks s(t) (columns W-AA).

$$s(t,k) = S(t,k) / P(t)$$
$$s(t) = S(t) / P(t)$$

Now we can have a look at the results. Table 1 shows the total stocks in 2008 in Mt in the use phase, obsolete stocks, and in landfills.

Table 1: Steel stocks, Germany 2008.

All in Mt	**Transport**	**Machinery**	**Buildings/Inf**	**Products**	**Total**
In Use	77	26	840	41	**980**
Obsolete	0	0	51	0	**51**
Landfills	68	26	69	110	**269**

Total stocks in 2008 were about 1 Gt, Another third of this amount can be found in obsolete stocks (old foundations and pipes, rests of tunnels or bridges, rusted away), and landfills.

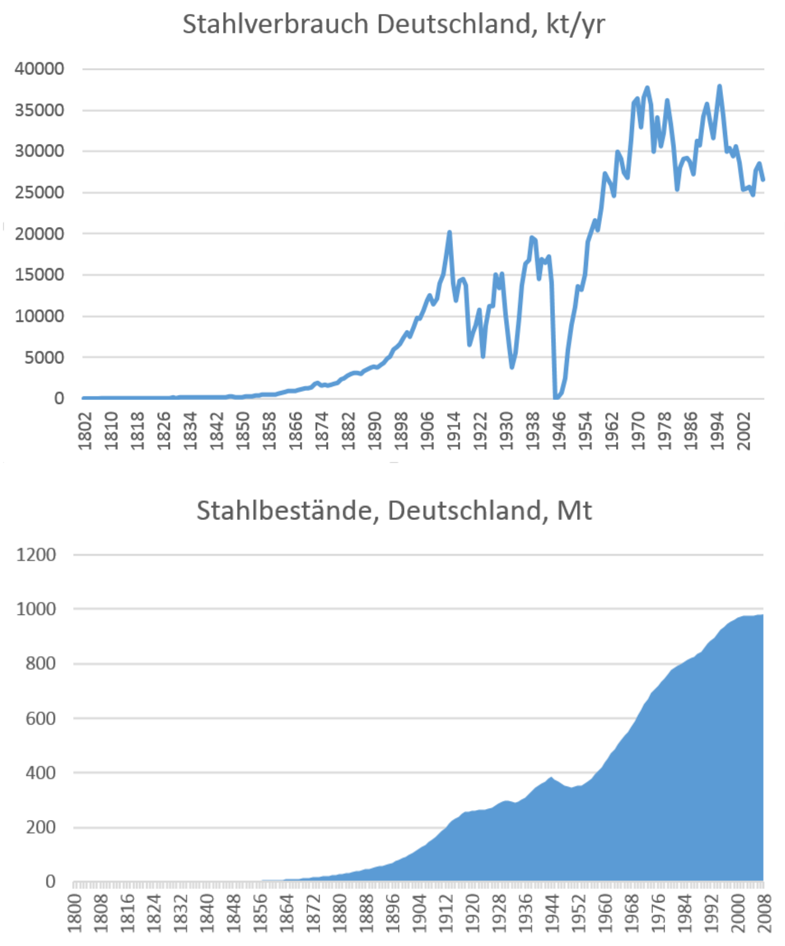

Fig. 3: steel consumption in kt/yr (top) and steel stocks in Mt (bottom), Germany, 1800-2008.

From Fig. 3 one can see that stocks are much less affected by economic crises and war than consumption flows. In 2008, stocks were about 30 times the size of the annual consumption. That means the people in Germany were using about 30 times the current annual production of steel in stocks, or it would take 30 years of current production levels to rebuild the steel stocks in use in 2008.

Finally, we want to look at steel stocks in different years (Table 2).

Table 2: per capita steel stocks, Germany.

All in Mt	Transport	Machinery	Buildings/Inf	Products	Total
1850	0.02	0.01	0.07	0.007	0.1
1900	0.3	0.2	1.1	0.1	1.7
1940	0.5	0.3	4.0	0.2	5.0
1970	1.0	0.4	5.9	0.5	7.8
1990	1.1	0.5	8.7	0.5	10.8
2008	0.9	0.3	10.2	0.5	11.9

Per capita steel stocks in 2008 were about 12 tons per person. The stocks in transport, machinery, and consumer products were stagnating or slightly declining, while stocks in buildings and infrastructure were still growing.