Industrial Ecology open online course (IEooc)

IEooc

Part II: Methods

Methodology 3: Dynamic Material Flow Analysis
IEooc_Methods3_Lecture2

Dynamic stock models

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Content

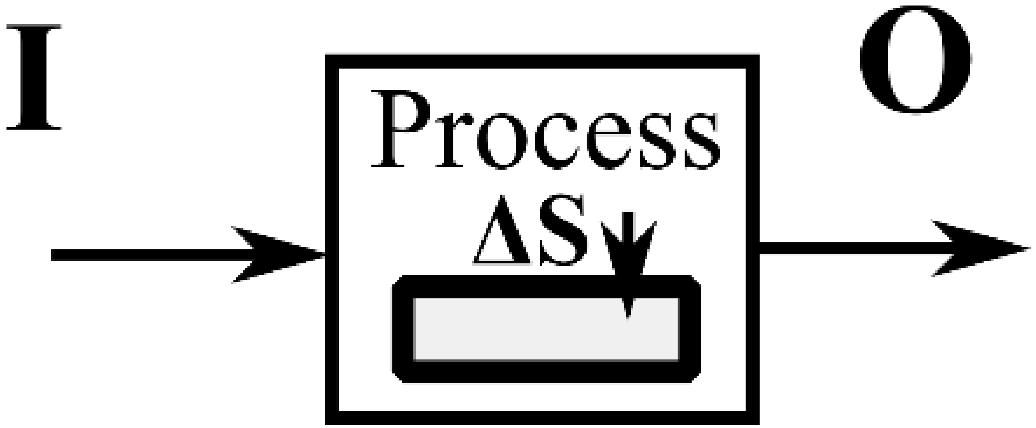
Population balance model ('Dynamic stock modelling')

The leaching model

Impulse response function, and age-cohorts

The lifetime model

Population balance model



$$\frac{dS(t)}{dt} = I(t) - O(t)$$

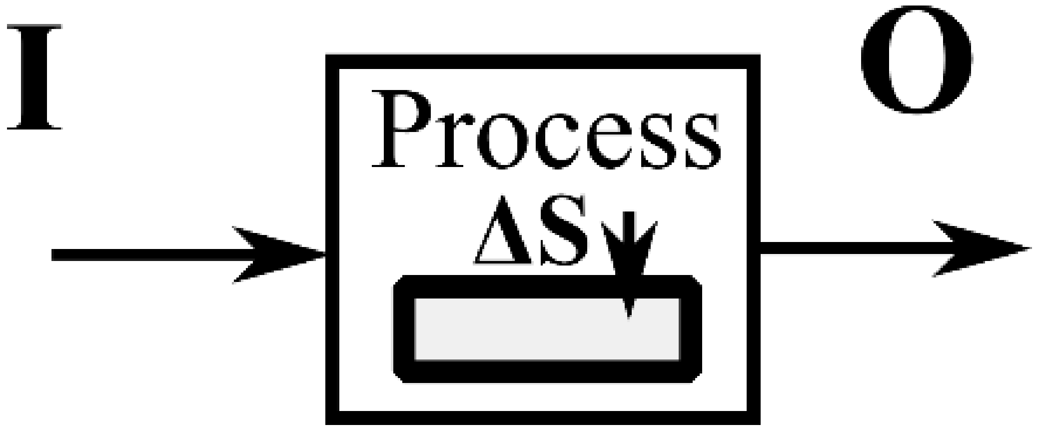
$$S(t) = \int_{t_0}^{t} \frac{dS(\tau)}{d\tau} d\tau$$

The mass balance equation dictates
The following relation between
Inflow, outflow, and stock change
of a process:

This relation applies to all processes
with a conservation law.

Non-conserved quantities can be
adopted: For a population of
individuals, I is the birth rate
and O is the death rate.

The leaching model



$$O(t) = c \cdot S(t)$$

At any given time, the outflow is proportional to the total stock present.

If no further inflow (I(t) = 0), the stock decays exponentially: $S(t) = S_0 \cdot \exp(-c \cdot t)$

Applications:

- Radioactive decay
- Leaching from deposits of pollutants or nutrients (landfills, tailings, soil)
- Simple to apply
- Only works for processes with no 'internal memory': probability of an item leaving the stock is independent of its residence time.

Van der Voet, Kleijn, et al. (2002): "Predicting future emissions based on characteristics of stocks", Ecological Economics, DOI 41 (2002) 223–234

https://en.wikipedia.org/wiki/Leaching_model_(soil)

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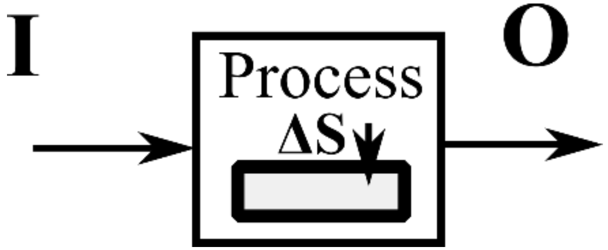
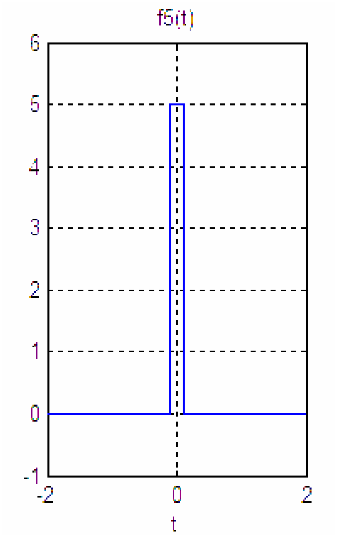
The impulse response function

The **impulse response**, or **impulse response function (IRF)**, of a dynamic system is its output when presented with a brief input signal, called an impulse.

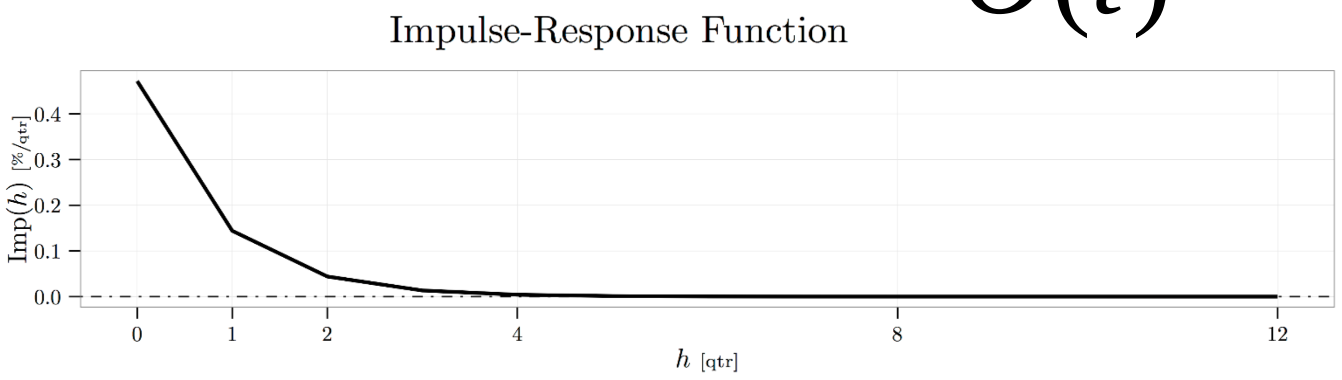


The impulse response function of a process with a stock is the outflow O(t) as response to an instantaneous inflow I(0) = I_0 at time t = 0.

$I(t)$



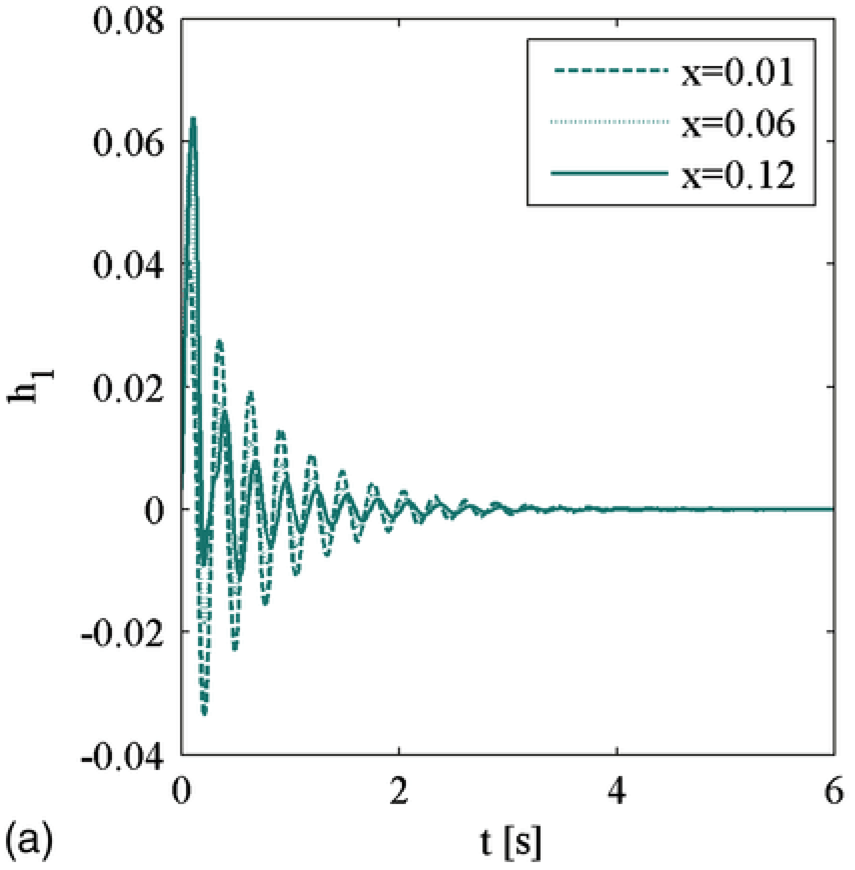
$O(t)$



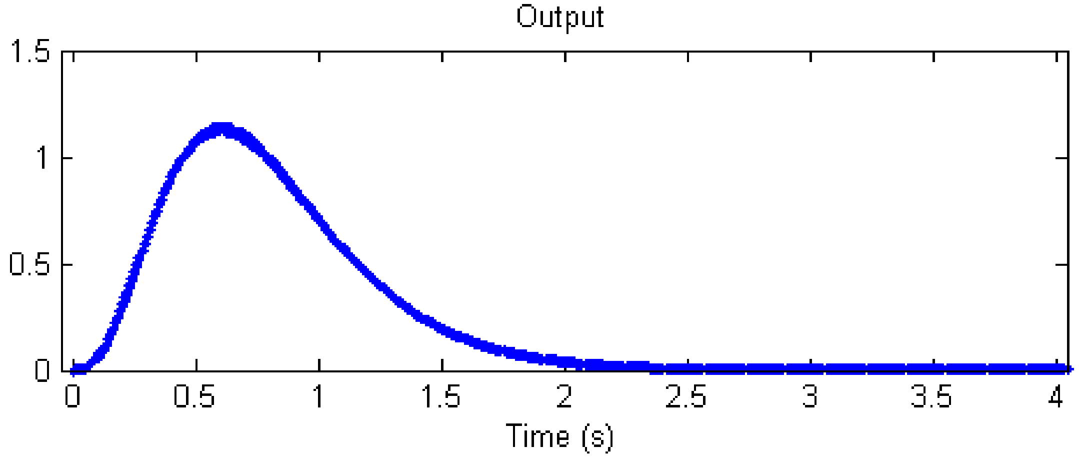
https://en.wikipedia.org/wiki/Impulse_response http://www.thefouriertransform.com/math/fnt2.gif
http://www.alexchinco.com/wp-content/uploads/2015/11/plot-cpi-ar1-irf1.png

Types of impulse responses

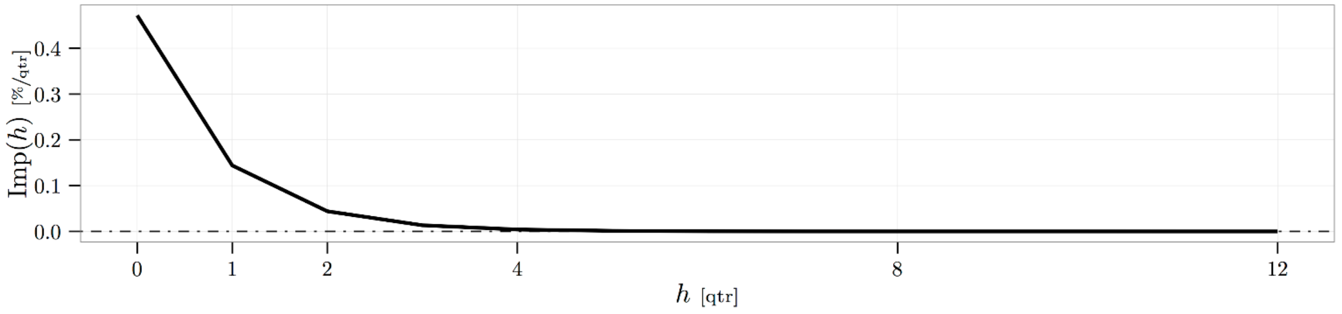
Oscillating



Delayed response



Decaying response



https://www.researchgate.net/profile/Marco_Broccardo2/publication/283683880/figure/fig2/AS:399452046151684@1472248197847/Fig-8-Impulse-response-function-white-noise-input.ppm
http://www.mbfys.ru.nl/~robvdw/DGCN22/PRACTICUM_2011/LABS_2011/ALTERNATIVE_LABS/Lesson_12_18.png
http://www.alexchinco.com/wp-content/uploads/2015/11/plot-cpi-ar1-irf1.png

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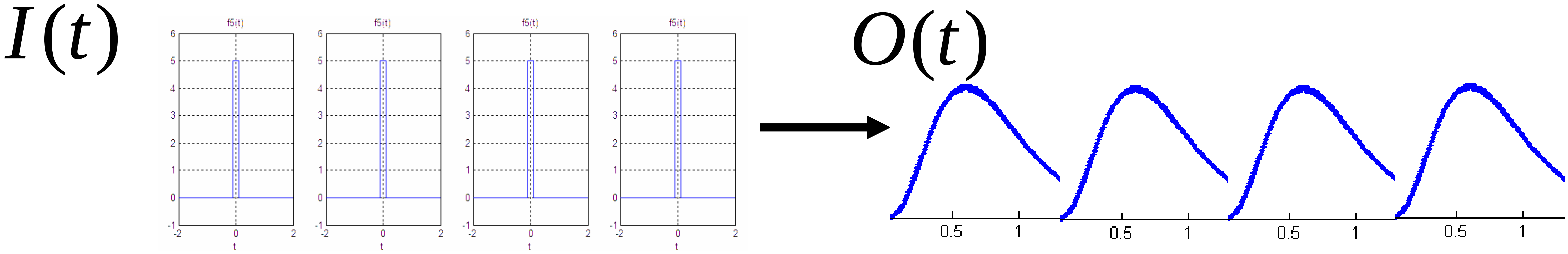
Response to an addition to stock: Age-cohorts and lifetime

Two types of response are commonly considered in dynamic stock modelling:

- Delayed response
- Decay

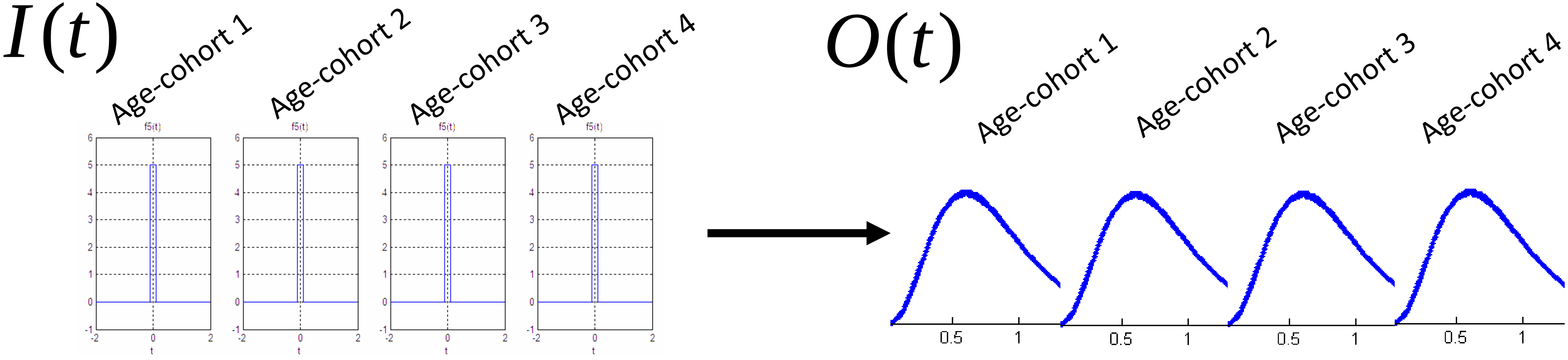
The response of the stock is linear: The responses to different input pulses are simply added up.
(analogy: The sound of an orchestra is the superposition (‘sum’) of the sounds of the different musical instruments.
Contrary: distortions added to guitar sounds in rock music



Response to an addition to stock: Age-cohorts and lifetime

For linear dynamic stock models (standard situation)

- Each input to stock can be traced separately, and the fraction of the stock that origins from a given input at time t is called the *age-cohort* (of) t.
- The different age-cohorts can be traced separately (individually).

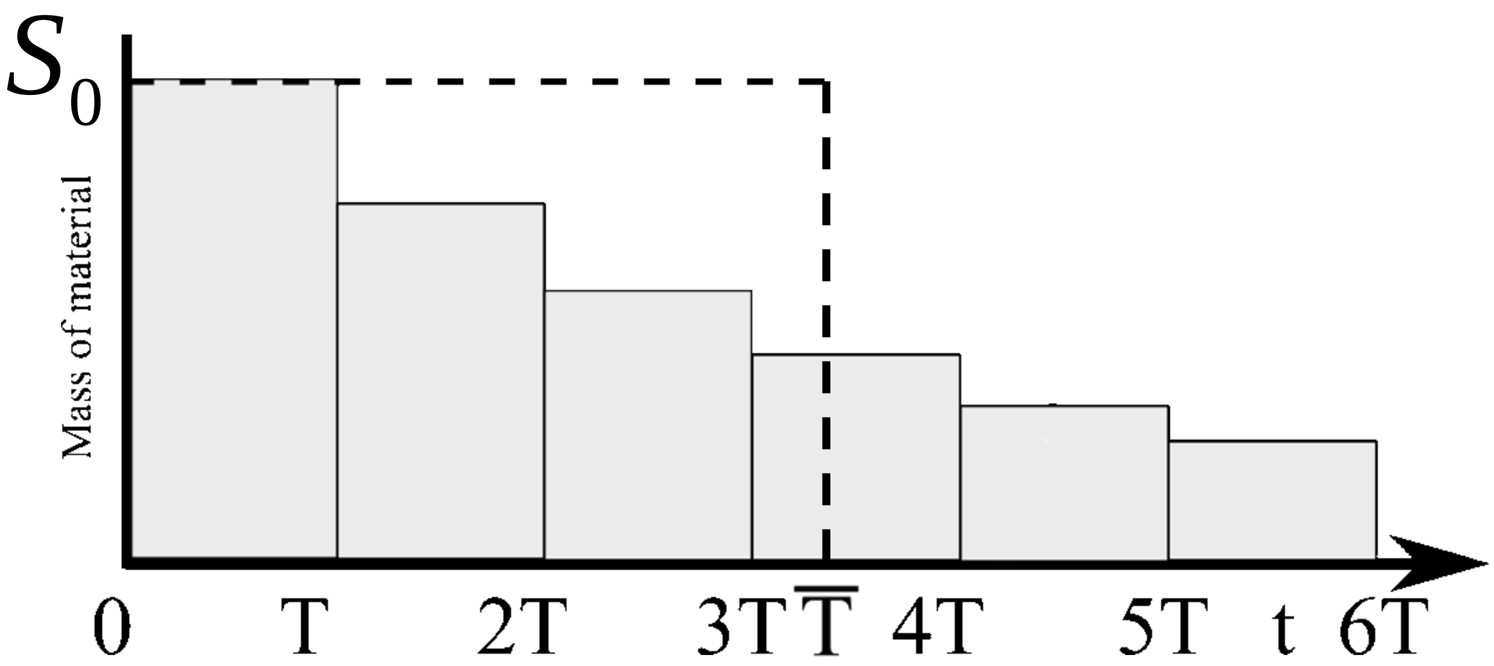


Response to an addition to stock: Age-cohorts and lifetime

The average lifetime $\bar{T}$ of an age-cohort is defined as the average residence time of the age-cohort in the stock.

Plot: Stock (t) after initial inflow at t=0.

$$\bar{T} = \frac{1}{S_0} \cdot \int_0^\infty S(t)dt$$



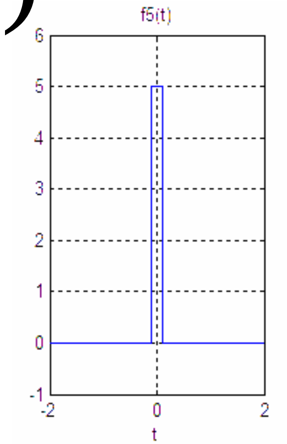
Read plot vertically: for each time t the stock at t is indicated.
Read plot horizontally: for the different fractions of the stock the total lifetime is indicated, sorted from shortest (top) to longest (bottom).
From combining both perspectives the average lifetime can be derived.

Fixed and distributed lifetimes

For a delayed response to an inflow (e.g. duration of product use) to cases are distinguished:

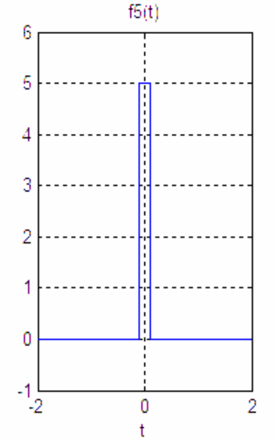
Fixed Product lifetime

$I(t)$



t=0

$O(t)$

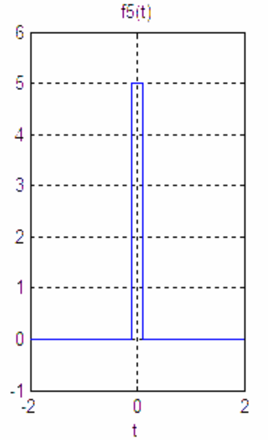


t=T

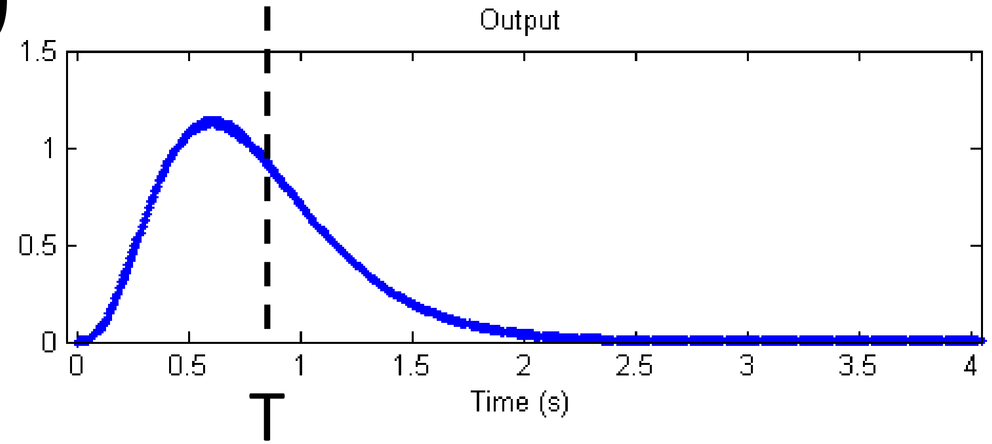
For fixed product lifetimes the entire age-cohort leaves the stock together after the lifetime T has expired. For the distributed lifetime the different fractions of the age-cohorts leave the stock with different Probabilities, so that the average lifetime (expectation value) equals T.

Distributed Product lifetime

$I(t)$



$O(t)$



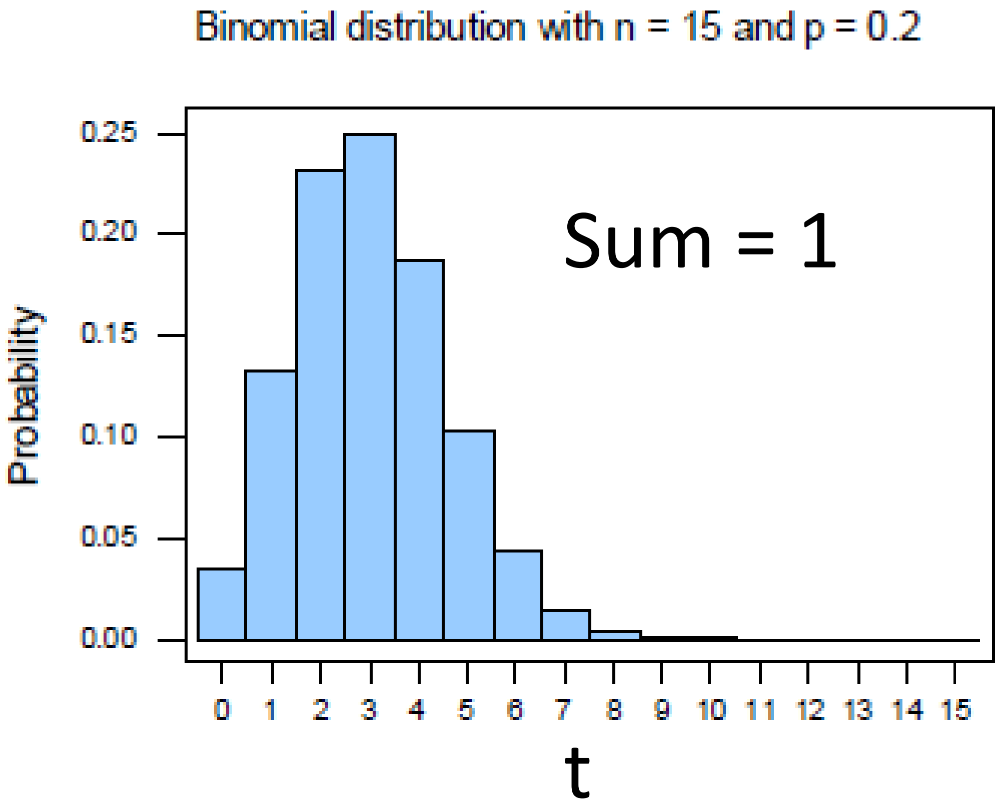
The probability distribution (discrete case)

Discrete probability distributions (*pd*) represent the probabilities for the different discrete events.

In dynamic stock modelling the events under consideration are that the product leaves the stock after time t, and the *pd* indicates probability of a product Becoming obsolete at time t.

For very large stocks (millions of cars etc.) the probability distribution indicates the fraction of the stock leaving.

Unit: 1



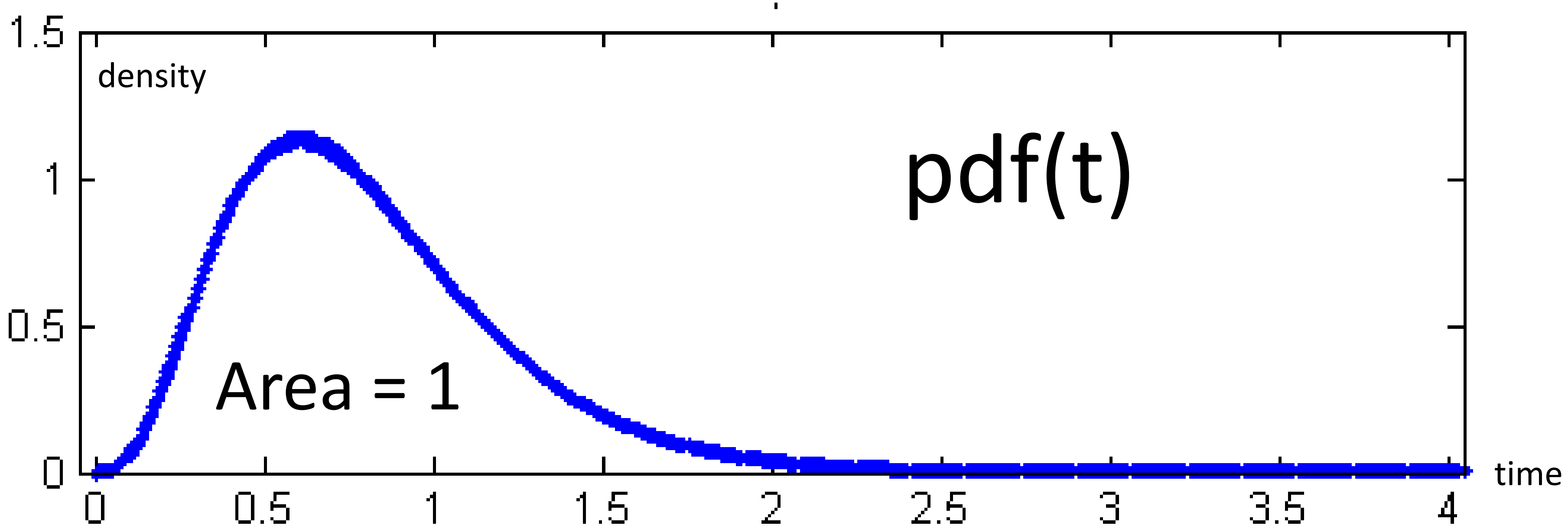
https://en.wikipedia.org/wiki/Probability_density_function
https://onlinecourses.science.psu.edu/stat414/sites/onlinecourses.science.psu.edu.stat414/files/lesson09/graph_n15_p02.gif

The probability density function (pdf)

Continuously distributed lifetimes are the norm in dynamic stock modelling, as they reflect reality best (age-cohorts of vehicles, buildings, electronic devices…)

If we take the outflow of an inflow pulse with a continuously distributed lifetime and normalize the curve so that the area is 1, we arrive at the *probability density function (pdf)* of the lifetime model.

Unit = 1/time
→ pdf(t) indicates rate of decay



https://en.wikipedia.org/wiki/Probability_density_function

The lifetime model for an input pulse

With the pf and the pdf we can recalculate the outflow of an input pulse as

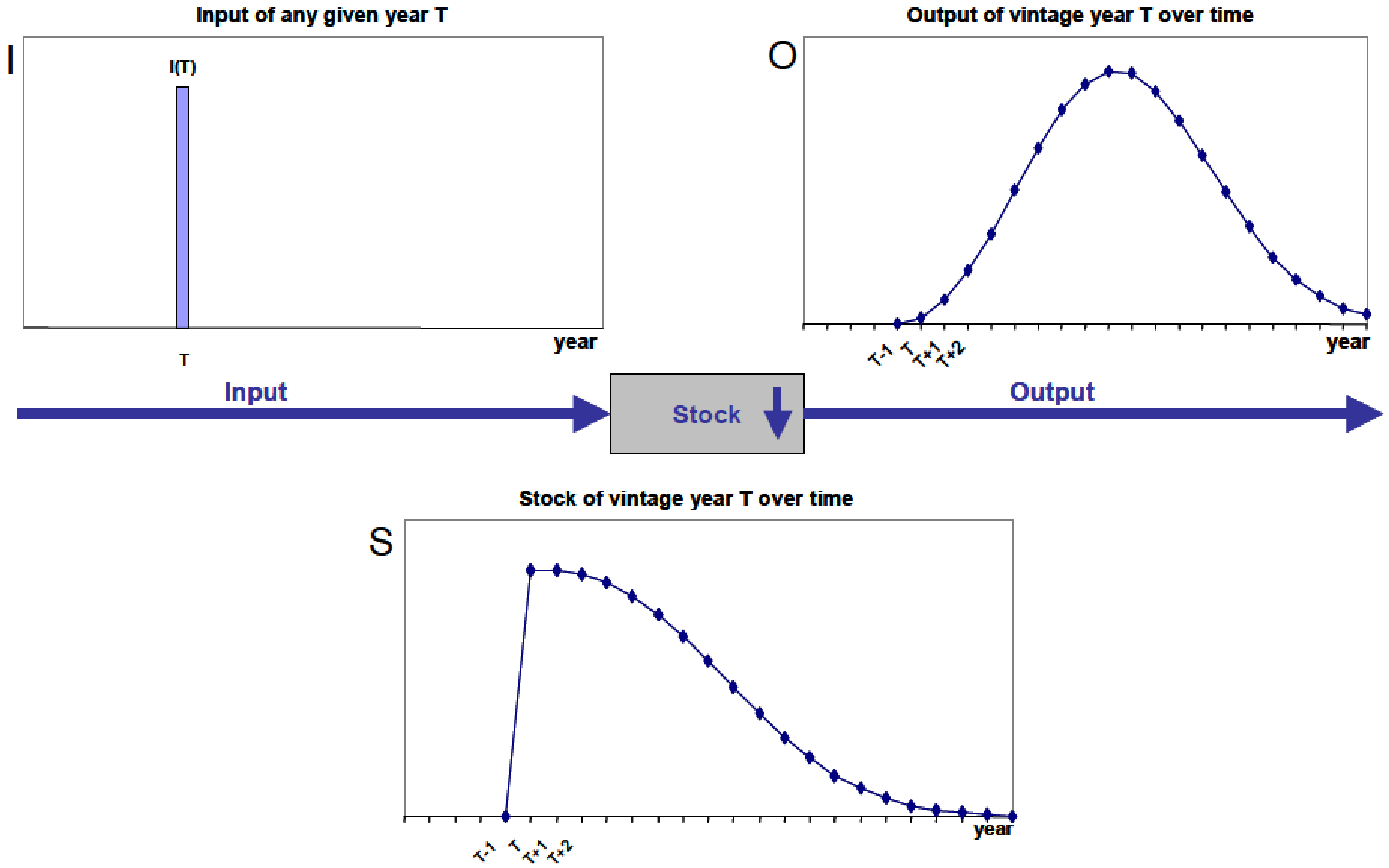
$$O(t) \qquad = I_0 \cdot pf(t - t_0)$$

$$[kt/yr] = [kt/yr] \cdot 1$$ (discrete case, I_0 is measured as a rate in kt/yr)

$$O(t) \qquad = I_0 \cdot pdf(t - t_0)$$

$$[kt/yr] = [kt] \cdot [1/yr]$$ (continuous case, I_0 is measured as amount in kt)

The lifetime model for an input time series



Müller et al. (2007): Service Lifetimes of Mineral End Uses. Report supported by U.S. Geological Survey (USGS), award number 06HQGR0174

The lifetime model for an input time series

In the case of a time series of input flows, we superpose (add) the outflows from the different age-cohorts:

$$O(t) \qquad = \sum_{t_0}^{t} I(\tau)\cdot pf(t-\tau)$$

$$[kt/yr] = [kt/yr]\cdot[1]$$ (discrete case, I(t) is measured as a rate in kt/yr)

$$O(t) \qquad = \int_{t_0}^{t} I(\tau)\cdot pdf(t-\tau)\,\mathrm{d}\tau$$

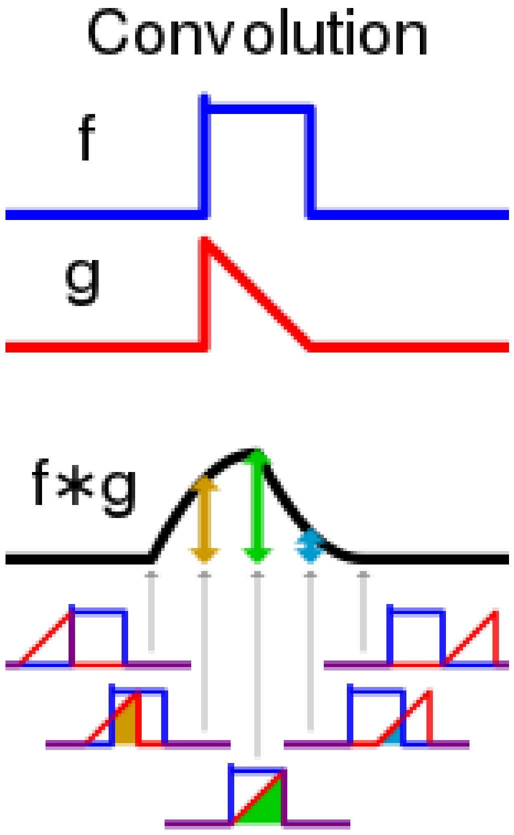
$$[kt/yr] = [kt/yr]\cdot[1/yr]\cdot[yr]$$ (continuous case, I(t) is also measured as a rate in kt/yr)

The lifetime model for an input time series

The mathematical operation used here is called *convolution*, which is the formal notion of applying a filter (here: delay from product lifetime) to an incoming signal.

$$O \quad = I * pdf$$

$$O(t) = \int_{t_0}^{t} I(\tau) \cdot pdf\,(t-\tau)\; d\tau$$



https://en.wikipedia.org/wiki/Convolution