Industrial Ecology Open Online Course (IEooc)

Part II Methods

Methods part 4 (Life cycle assessment)

http://www.teaching.industrialecology.uni-freiburg.de/

IEooc_Methods4_Exercise3: LCA on paper, matrix operations

Goal: *Life cycle perspective, understand the computational structure of LCA, understand and implement basic matrix operations*

Exercise part I, Reading:

As an introduction to the LCA method, read chapter 1 of the LCA textbook, which you find under the following link: https://cmu.app.box.com/s/5mnzyq1y3gcyjrveubf4/1/2746878222

Answer the end-of-chapter questions 1-5 in chapter 1 of the LCA textbook!

Exercise part II, small matrix operation tutorial:

LCA builds upon linear models of the industrial system, the environmental mechanisms, and the damage functions. The information about the coupling between the different parts of the LCA system is stored in tables, where the rows and columns stand for the different industrial processes, environmental mechanisms, or the interchanges (flows) between them. The dimensions (rows and columns) of all system variables and parameters in LCA will be explained in detail in Tuesday's lectures. Today, we practice how the system variables can be calculated step by step from final demand using matrix operations.

Example 1, multiplication of matrix with vector: Let **x** denote a set of industrial outputs, and **A** a table of the production functions ('recipes' or inter-industrial requirements) of the industries. Then one can determine the total amount of precursor products needed for the production of **x** by multiplying each column of A with the respective x entry (first column with first entry, second column with second entry, etc.) and summing up the respective contributions. The result is identical to the matrix multiplication defined as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad A \cdot x := \begin{bmatrix} a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 \\ a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 \\ a_{31} \cdot x_3 + a_{32} \cdot x_2 + a_{33} \cdot x_3 \end{bmatrix}$$

Task: Determine by hand and calculator the total amount of precursor products required for the production of the following output and with the production functions as follows. The row labels of **A** are: cars, steel, and electricity. The column labels of **A** are: car manufacturing, steel production, and electricity generation.

$$x = \begin{bmatrix} \text{1 car} \\ \text{100 tons of steel} \\ \text{3000 kWh of electricity} \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & 0 \\ \text{0.8 t/car} & \text{0.05 t/t} & 10^{-5}\,\text{t/kWh} \\ \text{2.7 MWh/car} & \text{22 kWh/t} & \text{0.1 kWh/kWh} \end{bmatrix}$$

Pay special attention to the units! What are the units of A·x?

Example 2, multiplication of matrix with matrix: Let **B** denote the emissions coefficients (emissions per unit of output) of the industrial processes, and **C** the coupling factors of emissions to environmental mechanisms (so-called characterization factors). From these two matrices one can determine the coupling of each sector to each environmental mechanism by multiplying each emissions factor with all corresponding characterization factors and by then summing up those pairs that belong to the same industry. The result is identical to the matrix multiplication of **B** and **C** defined as

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix},$$

$$C \cdot B := \begin{bmatrix} c_{11} \cdot b_{11} + c_{12} \cdot b_{21} + c_{13} \cdot b_{31} & c_{11} \cdot b_{12} + c_{12} \cdot b_{22} + c_{13} \cdot b_{32} & c_{11} \cdot b_{13} + c_{12} \cdot b_{23} + c_{13} \cdot b_{33} \\ c_{21} \cdot b_{11} + c_{22} \cdot b_{21} + c_{23} \cdot b_{31} & c_{21} \cdot b_{12} + c_{22} \cdot b_{22} + c_{23} \cdot b_{32} & c_{21} \cdot b_{13} + c_{22} \cdot b_{23} + c_{23} \cdot b_{33} \\ c_{31} \cdot b_{11} + c_{32} \cdot b_{21} + c_{33} \cdot b_{31} & c_{31} \cdot b_{12} + c_{32} \cdot b_{22} + c_{33} \cdot b_{32} & c_{31} \cdot b_{13} + c_{32} \cdot b_{23} + c_{33} \cdot b_{33} \end{bmatrix}, \qquad C \cdot B \neq B \cdot C$$

Task: Determine by hand and calculator the midpoint indicators per industrial process with the characterisation factors and emissions matrix as follows:

C: Characterization factors

Env. Mech. \ Emission:	CO_2 (t)	CH_4 (kg)	PM2.5 (g)
GWP (kg CO_2-eq)	1000 kg CO_2-eq/t	25 kg CO_2-eq/kg	0 kg CO2-eq/g
PM (kg)	0 kg/t	0 kg/kg	0.001 kg/g

B: Emissions by industries

Emiss. \ Industry:	Car manufact.	steel production	electricity generation
CO_2	0.8t/#	1.6 t/t	0.0006 t/kWh
CH_4	0.1 kg/#	7 kg/t	3e-3kg/kWh
PM2.5	8 g/#	270g/t	0.1 g/kWh

Pay special attention to the units! What are the units of C·B?

Example 3, matrix inversion and the unit matrix: Matrices are more complex objects than single numbers, and care has to be taken when performing matrix operations. For example, the product of two matrices is NOT independent of the order of the matrices, and matrix inversion is more complex than just computing 1 divided by each matrix element. The left inverse of a matrix is defined as follows:

$$M^{-1} \cdot M = I, \quad M^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1.2 & 2.5 & 1 & 0 \\ 0.24 & 0.5 & 0 & 1 \end{pmatrix}, \quad M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1.2 & -2.5 & 1 & 0 \\ -0.24 & -0.5 & 0 & 1 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Where the right matrix **I** is called unit matrix.

Task: Confirm that the above equation holds for the given M and its inverse M^{-1}!

Example 4, vector diagonalization and matrix row sum: Sometimes, it is practical to not sum over the different elements of a vector when determining emissions, for example. Summation can be avoided by moving all elements of a vector onto the diagonal of a matrix, as shown below:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \hat{x} := \begin{bmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{bmatrix}$$

The opposite operation, called the matrix row sum, can also be useful. It consists of the multiplication of a summation vector with only 1s from the right:

$$e := \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \hat{x} \cdot e = \begin{bmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Task: Compute on paper the terms A·x., A·x^hat, and A·x^hat·e, with A and x as defined above. What is the difference between A·x. and A·x^hat?

More help on matrix multiplication can be found here: https://www.mathsisfun.com/algebra/matrix-multiplying.html

Exercise part III: the computational structure of LCA on paper

Suppose you are hungry and want to eat a doughnut and a piece of cake in your local bakery. The bakery produces four types of products (intermediate products and final products that are sold to customers): cake (12 pieces of cake each), doughnut batches (50 pieces each), dough, and sugar coating. The production of one cake requires 1200 g of dough and 240 g sugar coating, and the production of one batch of doughnuts needs 2.5 kg of dough and 500 g of sugar coating.

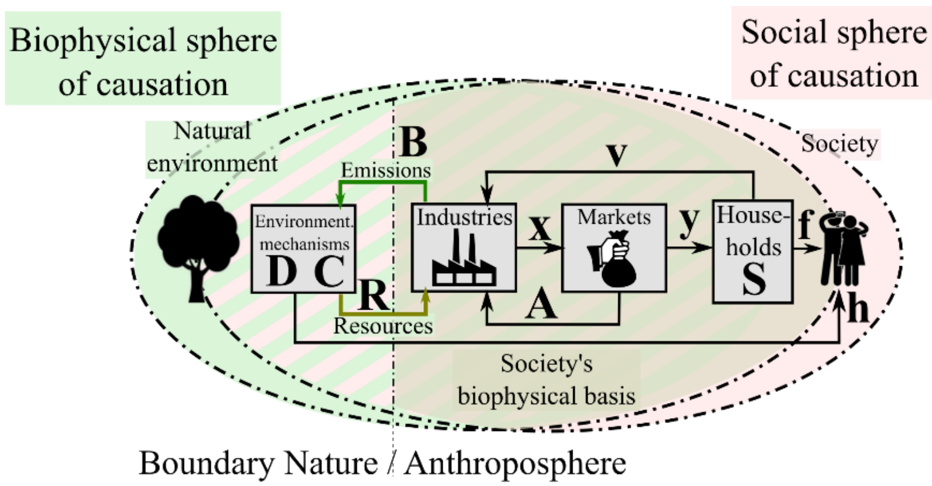

Figure 1: System structure of LCA.

Suppose further that two types of emissions occur in the bakery: Flour dust (10^-3 kg per kg of dough, causing particulate matter contamination, PM) and food colouring dust (10^-6 kg per kg of sugar coating), causing particulate matter contamination and exposure to allergenic substances.

Particulate matter contamination of any kind may eventually lead to asthma. Assume that the estimated health impact of asthma is 10^-5 DALY (disease-adjusted life years lost) per kg of particulate matter emitted, and the estimated impact of allergies is 10^-3 DALY per kg of food colouring emitted.

With that information perform the following tasks within the LCA framework:

Tasks:

1) How large are the particulate matter emissions and the exposure to allergenic substances associated with your hunger?
2) How large is the damage created (in DALY) from your hunger?
3) Convert the data given for the case study into the parameter framework of LCA (cf. Fig. 1)

$$h = D \cdot C \cdot B \cdot (I - A)^{-1} \cdot S \cdot f$$

What is the meaning of each symbol? Which dimensions do the different symbols have (in general: no. of products x no. of emissions, etc., and for this case: 2 x 4, 3 x 1, etc.)? Which of the parameters is a matrix and which one is a vector? Write down each parameter for the given case!

4) Bring your calculation into LCA format: Calculate, step by step,
   - The reference flow $y = S \cdot f$
   - The total bakery output $x = (I - A)^{-1} \cdot y$

- The total emissions $b = B \cdot x$
- The total environmental impact $c = C \cdot b$
- The total damage $h = D \cdot c$

5) Interpret your results! Which environmental mechanism causes the largest health damage? Which product causes the largest health damage? Who is likely to be most affected by the health damage?

Hint: If (and only if) the bakery products are ordered as follows: cake, doughnut, dough, sugar coating, the matrix (I-A)^-1 is as follows:

$$L = (I - A)^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1.2 & 2.5 & 1 & 0 \\ 0.24 & 0.5 & 0 & 1 \end{pmatrix}$$