

IEooc_Methods5_Exercise1: Basics of Input-Output Modelling

Goal: Introductory exercise for IO-analysis: Understand the mathematical basics of IO modelling, apply system structure of IO

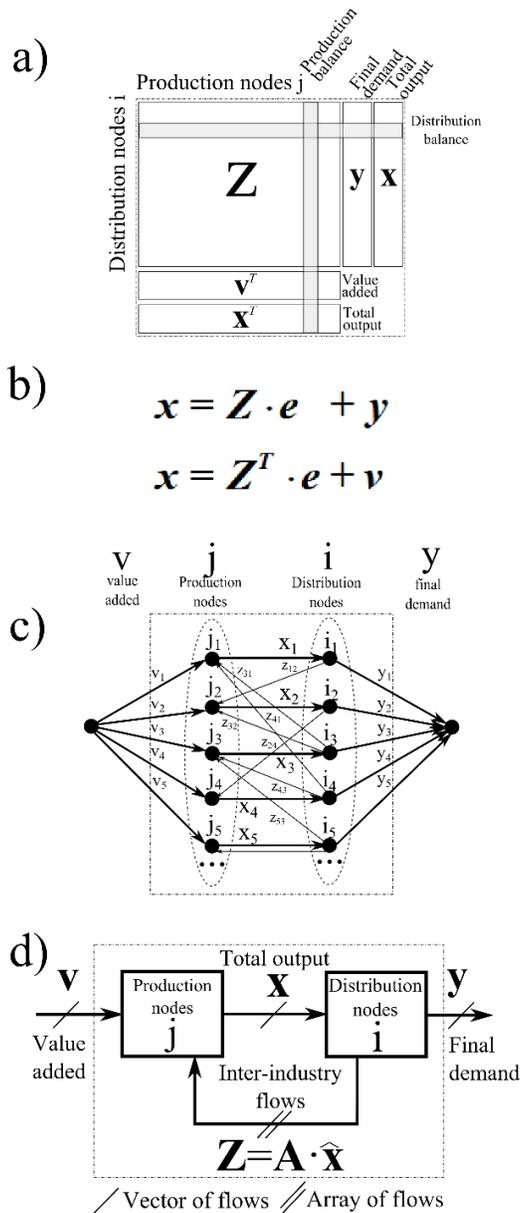


Fig. 1: Four equivalent representations of the IO-systems. a) tabular form, b) through accounting equations, c) as bipartite directed graph, d) process diagram.

Input-output analysis is based on a system description including both: transformation activities (industries, organisms) and distribution processes (markets). It contains four system variables: entry vector \mathbf{v} ('value added'), total output vector \mathbf{x} , final demand vector \mathbf{y} , and matrix of intermediate \mathbf{Z} (or $\mathbf{A} \cdot \mathbf{X}^\wedge$)

Exercises for model understanding:

- 1) Make sure that the four system descriptions of the IO-model as shown in figure 1 are equivalent i.e. every description can be transformed into another without loss of information. How?
- 2) What do the upper and lower formulas of fig. 1 mean? Describe their meaning using your own words!
- 3) How many processes are there in an IO-system with 18 industries and products?
- 4) How do you identify the matrix for production functions \mathbf{A} using the given system variables? What exactly is the meaning of a matrix entry for \mathbf{A} ? What does an \mathbf{A} column, \mathbf{A} row stand for?
- 5) **(Optional!)** How do you obtain the matrix of market shares \mathbf{B} using the given system variables? What exactly is the meaning of a matrix entry of \mathbf{B} ? What does a \mathbf{B} column, \mathbf{B} row stand for?
- 6) **(Optional!)** Prove that $\mathbf{A} = \hat{\mathbf{X}} \mathbf{B} \hat{\mathbf{X}}^{-1}$ and $\mathbf{B} = \hat{\mathbf{X}}^{-1} \mathbf{A} \hat{\mathbf{X}}$! What do these formulas stand for?
- 7) Determine the formula to calculate Leontief Inverse \mathbf{L} using given system variables! What does a single entry for \mathbf{L} stand for? What's the difference between \mathbf{A} and \mathbf{L} ?
- 8) **(Optional!)** Determine the formula to calculate Gosh inverse \mathbf{G} using given system variables! What does a single entry for \mathbf{G} stand for? What's the difference between \mathbf{B} and \mathbf{G} ?
- 9) **(Optional!)** Describe two specific examples for when to use Leontief and Gosh matrices. During the cold war era the hypothesis was that the Leontief model represents the capitalistic economic system while the Gosh model represents the socialistic planned economic system. How did people come up with this idea? Do you agree/disagree?

Exercise continues on next page.

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Numerical exercise:

In the table 'Raw Data' of the work book 'IEooc_Methods5_Exercise1_IO_Basics.xlsx' provided you can find data for a simple IO system with four industries and markets. The cells used to determine each step of the solution are already marked.

1. Complete the data base (blue): Calculate the value added of the processes to obtain the four system variables to meet the requirements for the market and process balance sheet!
2. Development of the IO-model (green): Calculate the A and L matrices for the given data! (Excel function: MMULT for matrix multiplication, MTRANS to transpose the matrices, MINV for matrix inversion. Caution! The results must be calculated using only matrix formulas. <https://support.office.com/en-us/article/Guidelines-and-examples-of-array-formulas-7d94a64e-3ff3-4686-9372-ecfd5caa57c7>)
3. What's the output vector x for given final demand y according to Leontief model? How big is the overall input v for the Leontief model and why is the overall input exactly that big?